

## THE NATURE OF WIND

### INTRODUCTION

**Definition of Wind** – Wind is often spoken of as “air in motion”. To one who looks at the atmosphere from the perspective of kinetic gas theory, this is “a distinction without a difference”. On the molecular level, air is always in motion, even under completely windless conditions. The various gaseous atmospheric molecules are eternally moving about. In doing so, they display a wide variety of speeds, both along their individual true paths and along their individual component paths.

Naturally, on the macroscopic level, these molecular motions are not usually perceptible to our bodily senses. Therefore, when we cannot sense any air movement, we say that “The air is still”. It is not still, of course, but—with our limited senses—we are simply unable to detect any perceptible movement.

Nevertheless, the main purpose of this collection of papers is to look at common atmospheric phenomena through the perspectives of kinetic gas theory and statistical mechanics. From those perspectives, the air is never still. It is *always* in motion.

Consequently, we will define wind as: **Wind is air in *net* motion.**

That is, when the wind is blowing, more air molecules have a component of motion in the wind direction than in any other direction. Moreover, their average speeds along this windward component of motion are somewhat faster than their average speeds along any other direction. In the discussions that follow, we will develop these two concepts in detail and at some length.

**Winds as Three-Dimensional Phenomena** – In this paper, when we speak of winds, we specifically include what some scholars separate out as air currents. That is, all of the winds that we will be discussing are three-dimensional phenomena.

When the wind blows across a landscape, it often follows the contours of that landscape—rising as the land rises and falling as the land slopes downward. Similarly, winds far above the ground have vertical components of motion as well as horizontal. Winds blowing across the crests of mountain ranges often form a series of standing waves that persist for significant periods of time. Thus, in including the vertical component of winds with the horizontal, we are simply being realistic. That is the way real winds move.

**Winds as Rivers of Air** – Winds are often compared to another common natural fluid phenomena—rivers. Like rivers, they flow with a variety of speeds—faster here and slower there. These slower wind speeds are often near their margins or where the winds flow across the ground or the sea. The faster speeds are usually (but not always) near the middles of their flows.

Like rivers, winds twist and turn, rarely ever following a straight line. Sometimes this is in response to the twists and turns of the landscape, sometimes it is in response to atmospheric conditions. The important thing to note in these fluid maneuverings is that these twists and turns can be up and down as well as side to side.

Unlike rivers, winds are not strongly constrained to seek any lower level. Although they are just as much subject to the force of gravity as are rivers, the mass density of moving air is much less than that of moving water. Hence, the resultant downward force is less and can be more easily overcome by other atmospheric forces. Winds can soar upwards for kilometers in the violent updrafts of cumulonimbi—and plunge downward just as readily in those same storm clouds. They are truly three-dimensional phenomena and these upward and downward components of the wind should never be overlooked.

Unfortunately, many wind measuring devices do ignore this third dimension.

**Wind Measurements** – Wind direction is generally indicated by a variety of devices that respond to the pressure differences created by the wind flow. These include weather vanes, windsocks, and balloons. There are also electronic devices that measure the movement of wind-borne aerosols. Some of these devices measure the true direction of the wind, but many only measure the horizontal components. The mathematical expressions developed in this paper only work with three-dimensional winds.

Finally, virtually all wind directions are averages over some period of time. Wind direction is just as variable as wind speed—sometimes even more so.

For our purposes,

**The wind direction is defined as that three-dimensional direction that enjoys the greatest flux of gaseous air molecules. The wind vector is a three-dimensional vector.**

**Variability in Wind Direction** – We must never lose sight of the fact that wind is the most variable of all meteorological phenomena. It is variable in both direction and in velocity. In the terminology of the *National Oceanic and Atmospheric Administration*, a wind can fluctuate in direction up to 60° over the space of only two minutes and **not** be considered “variable”. Given that perspective, you can imagine how much “variable” winds fluctuate in direction over that same period of time.

That two-minute time frame is what U. S. government agencies use. Most of the rest of the world uses ten minutes. Therefore, both direction and velocity must be considered averages—averages over time and averages over space.

Because of eddies, almost every wind contains small (or not so small) areas where—for some period of time—the local wind will be blowing in a direction that is completely opposite to the prevailing wind direction. Therefore, when we say the wind is blowing in a particular direction at a particular speed we are generalizing. We are describing averages, not persistent values. Moreover, we are describing averages that are made over a significantly broad actual range of values.

Before we can properly describe wind speeds and wind directions, however, we have to supplement our current notational system.

### **A THREE-DIMENSIONAL NOTATIONAL SYSTEM**

**Limitations of the One-Dimensional Notation System** – Our current notational system is a [\*one-dimensional notation system\*](#) with *p* and *i* subscripts to denote whether or not the molecule is simply moving toward our object of interest (*p*) or about to interact with it (*i*). The only directions that we are concerned with in this system are **toward** the object of interest and **away from** the object of interest. We need a second notation system to denote how that particular component of molecular motion relates to the wind direction.

To do this, we will postulate a standard tri-axial orthogonal reference system; that is, an *x*-axis, a *y*-axis and a *z*-axis—with all three axes at right angles to one another. This system does not replace our *p*-axis, it supplements it. Winds are three-dimensional phenomena, therefore we need a three-dimensional notation system to describe them. This same notational system is used in other papers in this collection that also refer to winds.

**Wind System Parameters** – In our discussion of winds, we will make use of the following parameters. Unless these terms bear a directional superscript or subscript as defined below, they are solely scalar terms. When they do bear such

a superscript or subscript, the parent terms are the scalar component of a vector expression.

- $\bar{\omega}$  Mean wind speed along the wind axis in meters per second.
- $\alpha$  Angle of incidence of the wind vector to the sensing surface in degrees. When the wind is normal to and toward that surface,  $\alpha$  is  $+90^\circ$ . When the wind is parallel to that surface,  $\alpha$  is  $0^\circ$ . When the wind is normal to and away from that surface,  $\alpha$  is  $-90^\circ$ . Intermediate angles of incidence will have intermediate values of  $\alpha$ . These values will range from  $+90^\circ$  through  $0^\circ$  to  $-90^\circ$ .
- $\bar{T}_i$  Mean temperature in Kelvins, measured normal to and toward the sensor.
- $\bar{p}_i$  Mean pressure in Pascals, measured normal to and toward the sensor.
- $\bar{n}$  Mean molecular number density in number of molecules per cubic meter.
- $\bar{m}_i$  Mean molecular impulse mass in kilograms. This is not the same as the mean molecular mass of a population of molecules. See [Molecular Masses](#) for the difference.
- $\bar{u}_p$  Mean component molecular kinetic energy of translation in joules, measured normal to and toward the sensor along a single axial arm.
- $\Sigma u_p$  The sum of those molecular kinetic energies of translation in joules, measured normal to and toward the sensor along a single axial arm.

- $\bar{v}$  Mean molecular speed in meters per second. Subscripts will indicate whether the term refers to the general population ( $p$ ) or the interactive sub-population ( $i$ ).
- $k_B$  Boltzmann's Constant in joules per molecule per Kelvin.
- $\bar{f}_i$  Mean molecular flux in number of molecules per square meter per second, measured normal to and toward the sensor.
- $\bar{\phi}_i$  Mean molecular impulse per impact in joules, measured normal to and toward the sensor.
- $\sigma$  *Sigma*. This is the axial root-mean-square molecular speed ( $v_p^{rms}$ ) in meters per second at temperature  $\bar{T}$  and mean molecular impulse mass  $\bar{m}_i$ . As such, it is also the standard deviation ( $\sigma$ ) of the axial molecular velocity distribution. As a speed and not a velocity, it is a scalar term, possessing only magnitude.

**Superscripts and subscripts: The following terms may be used as either subscripts or as superscripts, depending upon which position is available.**

- $p$  Denotes membership in that portion of the general population of molecules that has a component of movement normal to and toward the object of interest, usually a sensor.
- $i$  Denotes membership in that portion of the interactive sub-population of molecules whose next or current interaction is with the object of interest, usually a sensor.
- $\rightarrow$  Denotes that the molecules designated by the parent expression have a component of motion in the windward direction.

- ← Denotes that the molecules designated by the parent expression have a component of motion in the leeward direction.
- ↓ Denotes that the molecules designated by the parent expression have a component of motion normal to and toward the wind axis.
- A** Denotes that the parent expression applies to molecules in Area A of the area under the normal curve. In still air, these are molecules with a component of movement in the future windward direction (→). In moving air, this windward direction remains the same.
- B** Denotes that the parent expression applies to molecules in Area B of the area under the normal curve. In still air, these are molecules with a very slight component of movement in the future leeward direction (←). In moving air, these are molecules with a very slight component of movement in the windward (→) direction. They are thus “turncoat” molecules, changing their allegiance when the wind blows.
- C** Denotes that the parent expression applies to molecules in Area C of the area under the normal curve. In still air, these are molecules with a component of movement in the future leeward direction (←). In moving air, this leeward direction remains the same.

**Wind Speed** – Let us postulate an intangible plane of one square meter in area. The plane is oriented normal to the wind direction. Let us now count the number of molecules passing through that plane in the windward direction in one second. We also count the number passing through the plane in the leeward direction in that same period of time. We subtract this second number from the first. We now take this remainder and divide it by the number density of the moving air. The result is the wind speed. That is,

$$\bar{\omega} = \frac{\bar{f}_i^{\rightarrow} - \bar{f}_i^{\leftarrow}}{\bar{n}}$$

NOW01

Here,  $\bar{\omega}$  is the mean wind speed in meters per second,  $\bar{f}_i^{\rightarrow}$  is the mean windward molecular flux in number per square meter per second,  $\bar{f}_i^{\leftarrow}$  is the mean leeward molecular flux in number per square meter per second, and  $\bar{n}$  is the mean molecular number density of the moving air in number of molecules per cubic meter.

**Wind speed is defined as the net molecular flux along the wind axis divided by the mean molecular number density.**

In actual practice, wind speed is measured by a variety of anemometers, both mechanical and electronic. The mechanical ones actually measure wind pressures and convert those pressures into speeds. The wind speed, of course, is the scalar component of the wind velocity.

Once again, many of these instruments only measure the horizontal component of the true wind velocity. Their results, as a consequence of this selection, may not be used in our three-dimensional equations.

### SPATIAL REFERENCE SYSTEMS

**Moving Air with a Moving Reference System** – When the wind is blowing, the air is in net movement. There are obviously more air molecules moving in the wind direction than in any other direction. However, if we move our reference system right along with the mass of moving air and at the same speed and direction, then the numbers of molecules moving through the reference system is still the same in every direction, and the distributions of molecular speeds along all possible axes of molecular movement are still the same as they are in still air.

These are the parameters that we would measure if we were in the basket of a balloon drifting with the wind. We will have the same temperature as we would have in still air, because the drifting kinetic energies are the same as in still



air. We will have the same pressure, because there is no change in the frequency of impacts upon the moving plane surface or in the impulses transferred to that surface. And, of course, we will have the same number density because nothing has been done to change it. As far as our instruments could tell, we were in still air and not drifting with the wind.

From the standpoint of the distribution functions of molecular speeds, the scalar values of the means, modes, medians, and standard deviations of the molecules of the moving air relative to the moving reference system are in every way identical to the scalar values of the means, modes, medians, and standard deviations of the molecules in still air relative to a stationary reference system. This is a very important concept. It should always be in the forefront of your mind in dealing with winds.

**Moving Air with a Fixed Reference System** – The situation changes considerably if we keep our reference system fixed and let the air move through it. The number density still remains the same, because nothing has yet been done to change it. However, the distribution of relative molecular speeds has changed considerably.

The molecules with a “drifting” component of motion in the windward direction now have individual component speeds that have increased by the scalar value of the wind speed. The molecules with a “drifting” component of motion in the leeward direction now have individual component speeds that have decreased by that same value.

Moreover, some of the molecules will have changed their allegiance. Those molecules with a “drifting” speed in the leeward direction that is less than the wind speed will be seen by the stationary observer as now moving in the windward direction. Moving very slowly, it must be admitted, but moving in the windward direction just the same.

### THE UBIQUITOUS AND EMINENTLY USEFUL NORMAL CURVE

In still air, the distribution of molecular speeds is the same along each and every possible axis of movement. Being a combination of all of these axes, our  $\rho$  axis shares this distribution. Equation MSV09 from [Molecular Speeds and Velocities](#) gives this distribution as:

$$\frac{dn}{dv_p} = \frac{\bar{n}}{\sqrt{2\pi}} \sigma \exp - \frac{v_p^2}{2\sigma^2} \quad \text{NOW02}$$

This expression gives the number of molecules (out of population  $\bar{n}$ ) that will have a component speed along our  $\rho$  axial arm between  $v_{\pm p}$  and  $dv_{\pm p}$ .

Since the distribution is symmetrical, the mean axial value  $\bar{v}_{\pm p}$  is already zero (0). Thus, to normalize the distribution, we need only to set the value of the standard deviation ( $\sigma$ ) to one (1). That gives us:

$$\frac{dn}{dv_p} = \frac{\bar{n}}{\sqrt{2\pi}} \exp - \frac{v_p^2}{2} \quad \text{NOW03}$$

This expression is readily seen as that of a standard normal curve as applied to the molecular population  $\bar{n}$ .

What we now have is an asymptotic curve whose base is calibrated in standard deviations ( $\sigma$ ). In theory, this base reaches to infinity ( $\infty$ ) in each direction. In practice, almost the entire population of  $\bar{n}$  molecules will be contained between plus and minus four standard deviations. All speeds related to this curve and the area under it must be expressed in standard deviations. This includes the wind speed ( $\bar{w}/\sigma$ ).

The values of the ordinates along this curve, of course, still represent the probability that a molecule chosen at random will have the speed (in *sigmas*) denoted by that ordinate's value along the abscissa.

**Boundary Ordinates** – Next, let us go on to say that this populated curve will always be divided into three areal segments by two boundary ordinates. The first of these boundary ordinates is the ordinate representing the median. Whether the air is still or the wind is blowing, this median is in the middle of the curve and divides the population exactly in half. Its value along the abscissa is zero standard deviations ( $0/\sigma$ ) when the air is still; and is ( $\bar{\omega}/\sigma$ ) standard deviations when the wind is blowing.

The second boundary ordinate is the negative value of the wind speed ordinate ( $-\bar{\omega}/\sigma$ ). In still air, since there is no wind as yet and hence no wind speed, this ordinate represents the negative scalar value of the future wind speed. It is always to the left of the median ordinate.

It is very important to note that, for most atmospheric wind speeds, this ordinate will be only a small fraction of one standard deviation to the left of the median ordinate. Even the ordinate representing the defining wind speed in a Class Five hurricane (70 meters per second) will be located less than a fifth of a standard deviation to the left of the median ordinate. In doing wind analyses, the area between the wind-speed ordinate and the zero ordinate (Area B) is always a very, very small portion of the total area under the curve.

**Area A** – Area A extends from the median ordinate ( $0/\sigma$ ) to  $+\infty$  at the extreme right tip of the normal curve. In still air, this area contains the number of molecules out of the total population  $\bar{n}$  that have an instantaneous component of motion from left to right ( $\bar{n}_p^A = \frac{1}{2}\bar{n}$ ), the future windward direction. In moving air, it contains the same number of molecules and they are still moving from left to right, but each of these molecules now has a speed increment of  $\bar{\omega}$  in the windward direction. This increment applies to all values of  $v_p$  in Area A.

It should be noted that the mean molecular impulse speed ( $\bar{v}_i$ ) is **not** increased simply by the wind speed. This is because this impulse speed is not part of the  $dn/dv_p$  distribution function, but a measure of an entirely different speed distribution function. Instead,  $\bar{v}_i^A$  increases by a factor of  $\frac{\pi}{2}\bar{\omega}$ .

**Area B** – Area B includes only a very narrow slice of the area under the normal curve. Its right-hand boundary is the median ordinate ( $\bar{\omega}/\sigma$ ). Its left-hand boundary is the negative value of the wind-speed ordinate  $-(\bar{\omega}/\sigma)$ . In still air, this area contains those molecules out of population  $\bar{n}$  that have leeward molecular component speeds that are slower than the future wind speed. This sub-population number is given by the expression ( $\bar{n}_p^B = a\bar{n}$ ). However, as soon as the wind blows, each of these molecules will find themselves (relative to a stationary observer) having a very slight component of motion in the windward direction. Their number, of course, does not change.

In short, when the wind begins to blow, none of the scalar values of the Area B molecular parameters are changed in the slightest. The directional component of these vectors along the wind axis is simply reversed.

**Area C** – Area C extends from the negative wind-speed ordinate  $-(\bar{\omega}/\sigma)$  to  $-\infty$  at the extreme left tip of the normal curve. In still air, this area contains that portion of the molecules in population  $\bar{n}$  having an instantaneous component of motion from right to left (the future leeward direction) that were not already included in Area B ( $\bar{n}_p^C = \frac{1}{2}\bar{n} - a\bar{n} = b\bar{n}$ ). In moving air, it contains the same number of molecules, but these molecules have undergone a speed decrement of  $\bar{\omega}$  in the leeward direction.

**NOTE:** It is important to remember that the segregation of molecules into the three areas above only applies to our imaginary molecules under our imaginary normal curve. By imaginary, I mean “existing only in our minds”.

The real molecules of any portion of the real atmosphere are not in any way segregated. A molecules, B molecules, and C molecules are all randomly (but not uniformly) distributed in any volume of air. During any significant period of time each individual molecule will change its area designation billions of times. These are the realities of kinetic gas theory and statistical mechanics.

### AREA PARAMETERS IN STILL AIR

**The Imaginary Sensing Plane** – In still air, directions of molecular movement are still denoted in our one-dimensional reference system. The object of interest that generates these *i* and *p* subscripts is our imaginary sensing plane. This plane can be oriented at will at any angle relative to the future wind vector. Moreover, since it is imaginary, we will endow it with the ability to instantly measure—with perfect accuracy—all of the atmospheric parameters in which we might have an interest.

In comparing the parameters of still air molecules as they register upon both sensing surfaces of our imaginary plane, we shall examine ten specific parameters:  $(\bar{n}_p)(\bar{v}_p)(\bar{v}_i)(\bar{f}_i)(\bar{\phi}_i)(\bar{p}_i)(\bar{u}_p)(\bar{u}_i)(\bar{T})$  and  $(\Sigma u_p)$ . These parameters are all defined in the section above on **Wind System Parameters**.

Let us start with this imaginary sensing plane oriented normal to the wind direction. The wind axis is thus identical to our *p* axis. All terms with a *p* subscript have a component of motion normal to and toward the sensing plane. They must have a component of motion toward the sensing plane in order to be sensed. However, they may be moving in either the windward ( $\rightarrow$ ) or the leeward ( $\leftarrow$ ) direction, depending upon the superscript arrows.

**Table 1: Curve Area “A” Parameters**

In Still Air (→ )	Change (Δ)	In Wind (→ )
$\bar{n}_p^A = \frac{1}{2}\bar{n}$	$\Delta\bar{n}_p^A = 0$	$\bar{n}_p^{A\rightarrow} = \frac{1}{2}\bar{n}$
$\bar{v}_p^A = \bar{v}_p$	$\Delta\bar{v}_p^A = +\bar{\omega}$	$\bar{v}_p^{A\rightarrow} = \bar{v}_p + \bar{\omega}$
$\bar{v}_i^A = \frac{\pi}{2}\bar{v}_p$	$\Delta\bar{v}_i^A = +\frac{\pi}{2}\bar{\omega}$	$\bar{v}_i^{A\rightarrow} = \frac{\pi}{2}(\bar{v}_p + \bar{\omega})$
$\bar{f}_i^A = \frac{1}{2}\bar{n}\bar{v}_p$	$\Delta\bar{f}_i^A = \frac{1}{2}\bar{n}\bar{\omega}$	$\bar{f}_i^{A\rightarrow} = \frac{1}{2}\bar{n}\bar{v}_i^{A\rightarrow}$
$\bar{\phi}_i^A = \pi\bar{m}_i\bar{v}_p$	$\Delta\bar{\phi}_i^A = \pi\bar{m}_i\bar{\omega}$	$\bar{\phi}_i^{A\rightarrow} = 2\bar{m}_i\bar{v}_i^{A\rightarrow}$
$\bar{p}_i^A = \bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{p}_i^A = \frac{\pi}{2}\bar{n}\bar{m}_i(2\bar{v}_p\bar{\omega} + \bar{\omega}^2)$	$\bar{p}_i^{A\rightarrow} = \bar{n}\bar{m}_i\bar{v}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}$
$\bar{u}_p^A = \frac{1}{2}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{u}_p^A = \frac{1}{2}\bar{m}_i(2\bar{v}_p\bar{\omega} + \bar{\omega}^2)$	$\bar{u}_p^{A\rightarrow} = \frac{1}{2}\bar{m}_i\bar{v}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}$
$\bar{u}_i^A = \bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{u}_i^A = \bar{m}_i(2\bar{v}_p\bar{\omega} + \bar{\omega}^2)$	$\bar{u}_i^{A\rightarrow} = \bar{m}_i\bar{v}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}$
$\bar{T}^A = \frac{\bar{m}_i\bar{v}_p\bar{v}_i}{k_B}$	$\Delta\bar{T}^A = \frac{\bar{m}_i(2\bar{v}_p\bar{\omega} + \bar{\omega}^2)}{k_B}$	$\bar{T}^{A\rightarrow} = \frac{\bar{m}_i\bar{v}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}}{k_B}$
$\Sigma u_p^A = \frac{1}{4}\bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\Sigma u_p^A = \frac{\pi}{8}\bar{n}\bar{m}_i(2\bar{v}_p\bar{\omega} + \bar{\omega}^2)$	$\Sigma u_p^{A\rightarrow} = \frac{1}{4}\bar{n}\bar{m}_i\bar{v}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}$

1.  $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$ . See [Gas Laws in the Free Atmosphere](#).
2.  $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ . See [Molecular Speeds and Velocities](#).
3.  $\sigma^2 = k_B\bar{T}/\bar{m}_i = \bar{v}_p^2 = \bar{v}_p\bar{v}_i$ . See [Kinetic Energies of Translation](#).
4.  $a = erf(\bar{\omega}/\sigma)$ .  $b = erfc(\bar{\omega}/\sigma)$ .  $a + b = 0.5$ .

**Comments on Table 1:**

1. The Table is divided into three columns. The first column defines how the ten parameters are expressed in still air. The second column describes the increment that the first column parameters experience when the wind blows. The third column describes the values of the ten parameters in wind.
2. Derivation of these left-hand column expressions can be found in the appropriate papers in the [Basic Parameters](#) division of this collection.
3. The value of the mean molecular impulse mass ( $\bar{m}_i$ ) is a function of the gaseous composition of the parcel of air in question. Usually, this means that it is a function of the specific humidity or the mixing ratio, since water vapor is the most variable gaseous component in the free atmosphere.
4. All of the molecules in Area A have an axial component of movement that is in the future windward direction ( $\rightarrow$ ).
5. When the wind blows, this axial component of motion is incremented by the wind speed ( $\bar{\omega}$ ). By this I mean that each individual molecular axial speed component in the future windward direction is incremented. This has the effect, of course, of incrementing the mean values as well, and by the same amount.
6. This increment has the expected effect on all advanced parameters that contain the  $\bar{v}_p$  factor.
7. Note that the mean pressure ( $\bar{p}_i$ ) in Pascals is exactly four times the total axial kinetic energy of translation ( $\Delta\Sigma\bar{u}_p$ ) in joules. This is true in still air, in wind, and in the increment.

**Table 2: Curve Area “B” Parameters**

In Still Air ( $\leftarrow$ )	Change ( $\Delta$ )	In Wind ( $\rightarrow$ )
$\bar{n}_p^{B\leftarrow} = a\bar{n}$	$\leftarrow$ becomes $\rightarrow$	$\bar{n}_p^{B\rightarrow} = a\bar{n}$
$\bar{v}_p^{B\leftarrow} = \frac{1}{2}\bar{\omega}$	$\leftarrow$ becomes $\rightarrow$	$\bar{v}_p^{B\rightarrow} = \frac{1}{2}\bar{\omega}$
$\bar{v}_i^{B\leftarrow} = \frac{\pi}{4}\bar{\omega}$	$\leftarrow$ becomes $\rightarrow$	$\bar{v}_i^{B\rightarrow} = \frac{\pi}{4}\bar{\omega}$
$\bar{f}_i^{B\leftarrow} = \frac{1}{2}a\bar{n}\bar{\omega}$	$\leftarrow$ becomes $\rightarrow$	$\bar{f}_i^{B\rightarrow} = \frac{1}{2}a\bar{n}\bar{\omega}$
$\bar{\Phi}_i^{B\leftarrow} = \frac{\pi}{2}\bar{m}_i\bar{\omega}$	$\leftarrow$ becomes $\rightarrow$	$\bar{\Phi}_i^{B\rightarrow} = \frac{\pi}{2}\bar{m}_i\bar{\omega}$
$\bar{p}_i^{B\leftarrow} = \frac{\pi}{4}a\bar{n}\bar{m}_i\bar{\omega}^2$	$\leftarrow$ becomes $\rightarrow$	$\bar{p}_i^{B\rightarrow} = \frac{\pi}{4}a\bar{n}\bar{m}_i\bar{\omega}^2$
$\bar{u}_p^{B\leftarrow} = \frac{\pi}{16}\bar{m}_i\bar{\omega}^2$	$\leftarrow$ becomes $\rightarrow$	$\bar{u}_p^{B\rightarrow} = \frac{\pi}{16}\bar{m}_i\bar{\omega}^2$
$\bar{u}_i^{B\leftarrow} = \frac{\pi}{8}\bar{m}_i\bar{\omega}^2$	$\leftarrow$ becomes $\rightarrow$	$\bar{u}_i^{B\rightarrow} = \frac{\pi}{8}\bar{m}_i\bar{\omega}^2$
$\bar{T}^{B\leftarrow} = \frac{\pi}{8}\frac{\bar{m}_i\bar{\omega}^2}{k_B}$	$\leftarrow$ becomes $\rightarrow$	$\bar{T}^{B\rightarrow} = \frac{\pi}{8}\frac{\bar{m}_i\bar{\omega}^2}{k_B}$
$\Sigma u_p^{B\leftarrow} = \frac{\pi}{16}a\bar{n}\bar{m}_i\bar{\omega}^2$	$\leftarrow$ becomes $\rightarrow$	$\Sigma u_p^{B\rightarrow} = \frac{\pi}{16}a\bar{n}\bar{m}_i\bar{\omega}^2$

1.  $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$ . See [Gas Laws in the Free Atmosphere](#).
2.  $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ . See [Molecular Speeds and Velocities](#).
3.  $\sigma^2 = k_B\bar{T}/\bar{m}_i = \bar{v}_p^2 = \bar{v}_p\bar{v}_i$ . See [Kinetic Energies of Translation](#).
4.  $a = erf(\bar{\omega}/\sigma)$ .  $b = erfc(\bar{\omega}/\sigma)$ .  $a + b = 0.5$ .



**Comments on Table 2:**

1. All of the molecules in Area B have a component of movement along the wind axis that is leeward ( $\leftarrow$ ) in still air, but becomes windward ( $\rightarrow$ ) when the wind begins to blow. The scalar values of these wind axis parameters is unaffected.

2. The mean value of these Area B component molecular speeds is given in the table as  $(\frac{1}{2}\bar{\omega})$ . This is actually an approximation, and not an equality. At normal wind speeds, this approximation is very, very close to being accurate. The slope of the curve over this small range of speeds is negligible, and real-world wind speed measurements are not very precise.

3. It is highly unlikely that the standard precision of scientific measurement of three significant figures and the proper order of magnitude would be affected in the slightest by my changing the approximation symbol to an equal sign.

4. A second important assumption in Table 2 is that the equivalence ( $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ ) continues to be valid for this partial area of the normal curve. This is a far more shaky assumption than our first one. It assumes that the relationship between the two molecular speed distribution functions remains the same in a small portion of the speed distribution as it is in the entire distribution. This is probably not true, but the low numerical values of the Area B parameters lead to quite small mathematical errors for real-world values under normal wind speeds.

5. I leave it to others who are mathematically more adept than I am to derive more precise values for  $(\bar{v}_p^B)$  and  $(\bar{v}_i^B)$ .

6. Note that the mean pressure ( $\bar{p}_i$ ) in Pascals is exactly four times the total axial kinetic energy of translation ( $\Delta\Sigma\bar{u}_p$ ) in joules. This is true in still air, in wind, and in the increment.

Table 3, dealing with Area C, is much more complex.

**Table 3: Curve Area “C” Parameters**

In Still Air (←)	Change ( $\Delta$ )	In Wind (←)
$\bar{n}_p^C = b\bar{n}$	None	$\bar{n}_p^{C\leftarrow} = b\bar{n}$
$\bar{v}_p^C = \frac{\bar{v}_p - a\bar{\omega}}{2b}$	$\Delta\bar{v}_p^C = -\bar{\omega}$	$\bar{v}_p^{C\leftarrow} = \bar{v}_p^C - \bar{\omega}$
$\bar{v}_i^C = \frac{\pi\bar{v}_p - a\bar{\omega}}{4b}$	$\Delta\bar{v}_i^C = -\frac{\pi}{2}\bar{\omega}$	$\bar{v}_i^{C\leftarrow} = \bar{v}_i^C - \frac{\pi}{2}\bar{\omega}$
$\bar{f}_i^C = b\bar{n}\bar{v}_p^C$	$\Delta\bar{f}_i^C = -b\bar{n}\bar{\omega}$	$\bar{f}_i^{C\leftarrow} = b\bar{n}\bar{v}_p^C - b\bar{n}\bar{\omega}$
$\bar{\phi}_i^C = 2\bar{m}_i\bar{v}_i^C$	$\Delta\bar{\phi}_i^C = -\pi\bar{m}_i\bar{\omega}$	$\bar{\phi}_i^{C\leftarrow} = 2\bar{m}_i\bar{v}_i^C - \pi\bar{m}_i\bar{\omega}$
$\bar{p}_i^C = 2b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{p}_i^C = -\pi b\bar{n}\bar{m}_i\bar{\omega}^2$	$\bar{p}_i^{C\leftarrow} = 2b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C - \pi b\bar{n}\bar{m}_i\bar{\omega}^2$
$\bar{u}_p^C = \frac{1}{2}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{u}_p^C = -\frac{1}{2}\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)$	$\bar{u}_p^{C\leftarrow} = \frac{1}{2}\bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)$
$\bar{u}_i^C = \bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{u}_i^C = -\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)$	$\bar{u}_i^{C\leftarrow} = \bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)$
$\bar{T}^C = \frac{\bar{m}_i\bar{v}_p^C\bar{v}_i^C}{k_B}$	$\Delta\bar{T}^C = -\frac{\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)}{k_B}$	$\bar{T}^{C\leftarrow} = \frac{\bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)}{k_B}$
$\Sigma u_p^C = \frac{1}{2}b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\Sigma u_p^C = -\frac{\pi}{4}b\bar{n}\bar{m}_i\bar{\omega}^2$	$\Sigma u_p^{C\leftarrow} = \frac{1}{2}b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C - \frac{\pi}{4}b\bar{n}\bar{m}_i\bar{\omega}^2$

1.  $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$ . See [Gas Laws in the Free Atmosphere](#).
2.  $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ . See [Molecular Speeds and Velocities](#).
3.  $\sigma^2 = k_B\bar{T}/\bar{m}_i = \bar{v}_p^2 = \bar{v}_p\bar{v}_i$ . See [Kinetic Energies of Translation](#).
4.  $a = \text{erf}(\bar{\omega}/\sigma)$ .  $b = \text{erfc}(\bar{\omega}/\sigma)$ .  $a + b = 0.5$ .

**Comments on Table 3:**

1. Keep in mind that all of the molecular speeds along the wind axis in Area C are to leeward. Thus, when the wind speed  $\bar{\omega}$  is added to these speeds, they are arithmetically diminished, not augmented. This diminution is shown in the Table as  $-\bar{\omega}$ , although it is not physically possible to have a negative wind speed. As a general rule, for Area C molecules, when the wind blows, all parameters that have a component of molecular speed along the wind axis are diminished.

2. By definition, the mean of a distribution is the sum of the individual values, divided by the number of values. For Area C, this means that:

$$\bar{v}_p^C = \frac{\frac{1}{2}\bar{n}\bar{v}_p - \frac{1}{2}a\bar{n}\bar{\omega}}{\frac{1}{2}\bar{n} - an} \quad \text{NOW05}$$

The denominator can be replaced by:

$$\bar{v}_p^C = \frac{\frac{1}{2}\bar{n}\bar{v}_p - \frac{1}{2}a\bar{n}\bar{\omega}}{b\bar{n}} \quad \text{NOW06}$$

A little algebraic simplification gives us:

$$\bar{v}_p^C = \frac{\bar{v}_p - a\bar{\omega}}{2b} \quad \text{NOW07}$$

3. The rest of the derivations are straightforward, and follow the pattern set in the above equations.

4. Note that the mean pressure ( $\bar{p}_i$ ) in Pascals is exactly four times the total axial kinetic energy of translation ( $\Delta\Sigma\bar{u}_p$ ) in joules. This is true in still air, in wind, and in the increment.

## **WINDWARD AND LEEWARD PARAMETERS WHEN THE WIND BLOWS**

Having parameters for A, B, and C, we can now develop parameters for windward (A+B) and leeward (just C) surfaces when the wind blows, and examine the increments and decrements in the resulting expressions. The imaginary sensing plane continues as the object of interest, and is still normal to the wind direction.

A useful way to imagine the effects of wind on molecular speeds and directions is to imagine that—when the wind blows—the entire normal curve is moved to the right while maintaining its complete integrity—that is, it maintains both its exact shape and its exact area. Only its position relative to the abscissa is changed.

After shifting to the right, the apex ordinate (mode) of the curve now has the value of  $(\bar{\omega}/\sigma)$  instead of 0 along the abscissa. The ordinate for the former wind speed  $-(\bar{\omega}/\sigma)$  is now at the 0 mark on the abscissa. The entire distribution curve has been moved to the right without any change in its shape or area. This is what happens when the wind blows. And a very useful thing it is!

**A Moving Parcel of Air with Two Observers** – Let us consider a thought problem. We start by postulating a moving parcel of air. The parcel is moving from left to right at a mean wind speed  $(\bar{\omega})$ . The parcel contains two observers, one drifting with the wind in the basket of a balloon and the second stationary but nearby at the same elevation. Both observers have instruments with which to measure atmospheric parameters.

Because the drifting observer is moving with the same velocity as the wind, he perceives himself to be surrounded by still air. His instruments show the same values as those displayed for still air in the various tables. The stationary observer sees that same air as moving past him at the wind speed. His instruments show significant differences. These differences are displayed for “In Wind” expressions in the various tables.

**Table 4: Windward Parameters**

In Still Air	Change ( $\Delta$ )	In Wind ( $\rightarrow$ )
$\bar{n}_p = \frac{1}{2}\bar{n}$	$\Delta\bar{n}_p = a\bar{n}$	$\bar{n}_p^{\rightarrow} = \bar{n}_p^{A\rightarrow} + \bar{n}_p^{B\rightarrow} = \frac{1}{2}\bar{n} + a\bar{n}$
$\bar{v}_p = \left(\frac{2(k_B\bar{T})}{\pi\bar{m}_i}\right)^{\frac{1}{2}}$	$\Delta\bar{v}_p = \bar{v}_p^{\rightarrow} - \bar{v}_p$	$\bar{v}_p^{\rightarrow} = \frac{(\bar{n}_p^{A\rightarrow}\bar{v}_p^{A\rightarrow}) + (\bar{n}_p^{B\rightarrow}\bar{v}_p^{B\rightarrow})}{\bar{n}_p^{\rightarrow}}$
$\bar{v}_i = \frac{\pi}{2}\bar{v}_p$	$\Delta\bar{v}_i = \bar{v}_i^{\rightarrow} - \bar{v}_i$	$\bar{v}_i^{\rightarrow} = \frac{(\bar{n}_p^{A\rightarrow}\bar{v}_i^{A\rightarrow}) + (\bar{n}_p^{B\rightarrow}\bar{v}_i^{B\rightarrow})}{\bar{n}_p^{\rightarrow}}$
$\bar{f}_i = \bar{n}_p\bar{v}_p$	$\Delta\bar{f}_i = \bar{n}_p^{\rightarrow}\bar{v}_p^{\rightarrow} - \bar{n}_p\bar{v}_p$	$\bar{f}_i^{\rightarrow} = \bar{n}_p^{\rightarrow}\bar{v}_p^{\rightarrow}$
$\bar{\phi}_i = 2\bar{m}_i\bar{v}_i$	$\Delta\bar{\phi}_i = 2\bar{m}_i\bar{v}_i^{\rightarrow} - 2\bar{m}_i\bar{v}_i$	$\bar{\phi}_i^{\rightarrow} = 2\bar{m}_i\bar{v}_i^{\rightarrow}$
$\bar{p}_i = \bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{p}_i = 2\bar{n}_p^{\rightarrow}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow} - \bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\bar{p}_i^{\rightarrow} = 2\bar{n}_p^{\rightarrow}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow}$
$\bar{u}_p = \frac{1}{2}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{u}_p = \frac{1}{2}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow} - \frac{1}{2}\bar{m}_i\bar{v}_p\bar{v}_i$	$\bar{u}_p^{\rightarrow} = \frac{1}{2}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow}$
$\bar{u}_i = \bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\bar{u}_i = \bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow} - \bar{m}_i\bar{v}_p\bar{v}_i$	$\bar{u}_i^{\rightarrow} = \bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow}$
$\bar{T} = \frac{\bar{m}_i\bar{v}_p\bar{v}_i}{k_B}$	$\Delta\bar{T} = \frac{\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow} - \bar{m}_i\bar{v}_p\bar{v}_i}{k_B}$	$\bar{T}^{\rightarrow} = \frac{\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow}}{k_B}$
$\Sigma u_p = \frac{1}{4}\bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Delta\Sigma u_p = \frac{1}{2}\bar{n}_p^{\rightarrow}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow} - \frac{1}{4}\bar{n}\bar{m}_i\bar{v}_p\bar{v}_i$	$\Sigma u_p^{\rightarrow} = \frac{1}{2}\bar{n}_p^{\rightarrow}\bar{m}_i\bar{v}_p^{\rightarrow}\bar{v}_i^{\rightarrow}$

1.  $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$ . See [Gas Laws in the Free Atmosphere](#).
2.  $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ . See [Molecular Speeds and Velocities](#).
3.  $\sigma^2 = k_B\bar{T}/\bar{m}_i = \bar{v}_p^2 = \bar{v}_p\bar{v}_i$ . See [Kinetic Energies of Translation](#).
4.  $a = erf(\bar{\omega}/\sigma)$ .  $b = erfc(\bar{\omega}/\sigma)$ .  $a + b = 0.5$ .

**Table 5: Leeward Parameters**

In Still Air ( $\leftarrow$ )	Change ( $\Delta$ )	In Wind ( $\leftarrow$ )
$\bar{n}_p^C = b\bar{n}$	None	$\bar{n}_p^{\leftarrow} = b\bar{n}$
$\bar{v}_p^C = \frac{\bar{v}_p - a\bar{\omega}}{2b}$	$\Delta\bar{v}_p^C = -\bar{\omega}$	$\bar{v}_p^{\leftarrow} = \bar{v}_p^C - \bar{\omega}$
$\bar{v}_i^C = \frac{\pi\bar{v}_p - a\bar{\omega}}{4b}$	$\Delta\bar{v}_i^C = -\frac{\pi}{2}\bar{\omega}$	$\bar{v}_i^{\leftarrow} = \bar{v}_i^C - \frac{\pi}{2}\bar{\omega}$
$\bar{f}_i^C = b\bar{n}\bar{v}_p^C$	$\Delta\bar{f}_i^C = -b\bar{n}\bar{\omega}$	$\bar{f}_i^{\leftarrow} = b\bar{n}\bar{v}_p^C - b\bar{n}\bar{\omega}$
$\bar{\Phi}_i^C = 2\bar{m}_i\bar{v}_i^C$	$\Delta\bar{\Phi}_i^C = -\pi\bar{m}_i\bar{\omega}$	$\bar{\Phi}_i^{\leftarrow} = 2\bar{m}_i\bar{v}_i^C - \pi\bar{m}_i\bar{\omega}$
$\bar{p}_i^C = 2b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{p}_i^C = -\pi b\bar{n}\bar{m}_i\bar{\omega}^2$	$\bar{p}_i^{\leftarrow} = 2b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C - \pi b\bar{n}\bar{m}_i\bar{\omega}^2$
$\bar{u}_p^C = \frac{1}{2}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{u}_p^C = -\frac{1}{2}\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)$	$\bar{u}_p^{\leftarrow} = \frac{1}{2}\bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)$
$\bar{u}_i^C = \bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\bar{u}_i^C = -\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)$	$\bar{u}_i^{\leftarrow} = \bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)$
$\bar{T}^C = \frac{\bar{m}_i\bar{v}_p^C\bar{v}_i^C}{k_B}$	$\Delta\bar{T}^C = -\frac{\bar{m}_i\left(\frac{\pi}{2}\bar{v}_p\bar{\omega} + \bar{v}_i\bar{\omega} - \frac{\pi}{2}\bar{\omega}^2\right)}{k_B}$	$\bar{T}^{\leftarrow} = \frac{\bar{m}_i\left(\bar{v}_p^C - \bar{\omega}\right)\left(\bar{v}_i^C - \frac{\pi}{2}\bar{\omega}\right)}{k_B}$
$\Sigma u_p^C = \frac{1}{2}b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C$	$\Delta\Sigma u_p^C = -\frac{\pi}{4}b\bar{n}\bar{m}_i\bar{\omega}^2$	$\Sigma u_p^{\leftarrow} = \frac{1}{2}b\bar{n}\bar{m}_i\bar{v}_p^C\bar{v}_i^C - \frac{\pi}{4}b\bar{n}\bar{m}_i\bar{\omega}^2$

1.  $\bar{p} = \bar{n}k_B\bar{T} = \bar{n}\bar{m}_i\sigma^2$ . See [Gas Laws in the Free Atmosphere](#).
2.  $\bar{v}_i = \frac{\pi}{2}\bar{v}_p$ . See [Molecular Speeds and Velocities](#).
3.  $\sigma^2 = k_B\bar{T}/\bar{m}_i = \bar{v}_p^2 = \bar{v}_p\bar{v}_i$ . See [Kinetic Energies of Translation](#).
4.  $a = \text{erf}(\bar{\omega}/\sigma)$ .  $b = \text{erfc}(\bar{\omega}/\sigma)$ .  $a + b = 0.5$ .

**Comments on Tables 4 and 5:**

1. The last step in our consideration of population areas under the wind curve is to combine the datasets for Columns A and B to produce a set of values for the total windward sub-population. This is done in Table 4.

We then rename the Area C parameters to become the leeward sub-population. This is done in Table 5.

2. We can now rewrite NOW01 to read:

$$\bar{f}_i^{\rightarrow} - \bar{f}_i^{\leftarrow} = \bar{n}\bar{\omega} \quad \text{NOW08}$$

3. In terms of our curve area parameters, this would be:

$$\bar{f}_i^{A\rightarrow} + \bar{f}_i^{B\rightarrow} - \bar{f}_i^{C\leftarrow} = \bar{n}\bar{\omega} \quad \text{NOW09}$$

4. Expanding the left-hand terms from the table values gives us:

$$\left(\frac{1}{2}\bar{n}(\bar{v}_p + \bar{\omega}) + \frac{1}{2}a\bar{n}\bar{\omega}\right) - \left(\frac{1}{2}(\bar{n}\bar{v}_p - a\bar{n}\bar{\omega}) - b\bar{n}\bar{\omega}\right) = \bar{n}\bar{\omega} \quad \text{NOW10}$$

5. Removing the parentheses leads to:

$$\frac{1}{2}\bar{n}\bar{v}_p + \frac{1}{2}\bar{n}\bar{\omega} + \frac{1}{2}a\bar{n}\bar{\omega} - \frac{1}{2}\bar{n}\bar{v}_p + \frac{1}{2}a\bar{n}\bar{\omega} + b\bar{n}\bar{\omega} = \bar{n}\bar{\omega} \quad \text{NOW11}$$

6. A little algebraic simplification results in:

$$\frac{1}{2}\bar{n}\bar{\omega} + a\bar{n}\bar{\omega} + b\bar{n}\bar{\omega} = \bar{n}\bar{\omega} \quad \text{NOW12}$$

7. Recognizing that  $a + b = \frac{1}{2}$ , leads to the equivalence:

$$\bar{n}\bar{\omega} = \bar{n}\bar{\omega} \quad \text{NOW13}$$

8. NOW08 and NOW13 both give the same values. Our table values are validated—at least in terms of the mean molecular component flows.

9. Note that the mean pressure ( $\bar{p}_i$ ) in Pascals is exactly four times the total axial kinetic energy of translation ( $\Delta\Sigma\bar{u}_p$ ) in joules. This is true in still air, in wind, and in the increment.

### Kinetic Energy Budgets along the Wind Axis

At this point, it is useful to focus on these kinetic energy budgets. Let us look more closely at how these totals change when the wind starts to blow.

When the wind starts to blow, all of the molecules with a windward component of motion undergo an increase in their total kinetic energy of translation in the windward direction. Tables 1 and 2 show this total windward increase in joules per cubic meter as:

$$\Delta\Sigma u_p^{\rightarrow} = \left( \frac{\pi}{8} \bar{n} \bar{m}_i \left( 2\bar{v}_p \bar{\omega} + \bar{\omega}^2 \right) \right) + \left( \frac{1}{2} a \bar{n} \bar{m}_i \bar{\omega}^2 \right) \quad \text{NOW14}$$

This can be rewritten as:

$$\Delta\Sigma u_p^{\rightarrow} = \frac{\pi}{4} \bar{n} \bar{m}_i \bar{v}_p \bar{\omega} + \frac{\pi}{8} \bar{n} \bar{m}_i \bar{\omega}^2 + \frac{1}{2} a \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW15}$$

At the same time, all of the molecules with a leeward component of motion suffer a decrease in their total kinetic energy of translation in the leeward direction. Table 3 shows this total decrease in joules per cubic meter as:

$$\Delta\Sigma u_p^{\leftarrow} = \Delta\Sigma u_p^C = -\frac{\pi}{4} b \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW16}$$

This can be rewritten as:

$$\Delta\Sigma u_p^{\leftarrow} = -\frac{\pi}{8} \bar{n} \bar{m}_i \bar{\omega}^2 + \frac{\pi}{4} a \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW17}$$



Adding NOW15 and NOW17 and performing a little algebraic simplification gives us the total change in kinetic energy of translation along the wind axis when the wind blows:

$$\Delta\Sigma u_p^{\leftrightarrow} = \frac{\pi}{4} \bar{n} \bar{m}_i \bar{v}_p \bar{\omega} + \frac{1}{2} a \bar{n} \bar{m}_i \bar{\omega}^2 + \frac{\pi}{4} a \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW18}$$

As Bernoulli pointed out centuries ago, when the wind blows, the kinetic energy of the system increases in proportion to the square of the wind speed.

The question now becomes, where does this extra energy come from? The observer drifting with the wind measures no increase in total kinetic energy in his enveloping parcel of air, so the increase must come from internal sources. The obvious sources (which Bernoulli anticipated) are the kinetic energies of the other four axial arms of movement. When the wind blows, each of these energies is diminished by a quarter of the amount in NOW18. That is:

$$\Delta\Sigma u_p^{\downarrow} = -\frac{\pi}{16} \bar{n} \bar{m}_i \bar{v}_p \bar{\omega} + \frac{1}{8} a \bar{n} \bar{m}_i \bar{\omega}^2 + \frac{\pi}{16} a \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW19}$$

This diminution has predictable consequences on the various parameters sensed by a sensor normal to those directions; i. e., by a sensor parallel to the wind direction.

In particular—keeping in mind the rigid relationship between the pressure and the total axial kinetic energy of translation—it enables us to state:

$$\Delta\bar{p}_i^{\downarrow} = \frac{\pi}{4} \bar{n} \bar{m}_i \bar{v}_p \bar{\omega} + \frac{1}{2} a \bar{n} \bar{m}_i \bar{\omega}^2 + \frac{\pi}{4} a \bar{n} \bar{m}_i \bar{\omega}^2 \quad \text{NOW20}$$

### **Parameters Measured by a Sensor Parallel to the Wind**

At this point, we can abandon our normal curve and the sub-populations under it. It is only useful when our imaginary sensor is normal to the windward-

leeward axis. Let us now turn that imaginary sensor until it is parallel to the wind direction. Later, we will see what happens at intermediate angles.

When the air is still, the number of molecules having a component of movement normal to and toward a sensor parallel to the future wind direction is  $\frac{1}{2}\bar{n}$ . This number does not change when the wind starts to blow. Therefore,

$$\bar{n}_p^\downarrow = \frac{1}{2}\bar{n} \quad \text{NOW21}$$

The mean component kinetic energy of translation decrement experienced by each of the above molecules is the total decrement divided by the number of molecules having that component of movement:

$$\Delta\bar{u}_p^\downarrow = \frac{\Delta\Sigma u_p^\downarrow}{\frac{1}{2}\bar{n}} = -\frac{\pi}{8}\bar{m}_i\bar{v}_p\bar{\omega} - \frac{1}{4}a\bar{m}_i\bar{\omega} - \frac{\pi}{8}a\bar{m}_i\bar{\omega}^2 \quad \text{NOW22}$$

This enables us to calculate the temperature decrement induced on an exposed surface parallel to the wind. Firstly,

$$\Delta\bar{u}_i^\downarrow = 2\Delta\bar{u}_p^\downarrow \quad \text{NOW23}$$

Hence,

$$\Delta\bar{T}^\downarrow = -\frac{\frac{\pi}{4}\bar{m}_i\bar{v}_p\bar{\omega} + \frac{1}{2}a\bar{m}_i\bar{\omega} + \frac{\pi}{4}a\bar{m}_i\bar{\omega}^2}{k_B} \quad \text{NOW24}$$

Having the decrements in both the pressure and the temperature induced by wind on surfaces parallel to the wind direction is sufficient for our present purposes. Those who have an interest in the other parameters normal to the wind direction may readily calculate them using the models already shown in this paper.

### Parameters Sensed by Sensors at Intermediate Wind Angles

The angle of incidence **alpha** ( $\alpha$ ) is the angle that the wind vector makes with the sensing surface. When the wind is normal to and toward the surface,  $\alpha$  is  $+90^\circ$ . When the wind is parallel to the surface,  $\alpha$  is  $0^\circ$ . When the wind is normal to and away from the surface,  $\alpha$  is  $-90^\circ$ . Intermediate angles will have intermediate values of  $\alpha$ , ranging from  $+90^\circ$  through  $0^\circ$  to  $-90^\circ$ .

When the angle of incidence is positive, the parameter ( $X$ ) sensed by the incident sensing surface is given by:

$$X^\alpha = (\sin \alpha) X^{\rightarrow} + (1 - \sin \alpha) X^{\downarrow} \quad \text{NOW25}$$

When the angle of incidence is negative, the parameter is given by:

$$X^\alpha = (\sin \alpha) X^{\leftarrow} + (1 - \sin \alpha) X^{\downarrow} \quad \text{NOW26}$$

**A Note on Scale** – The equations in this paper assume that the wind flows discussed are free of aberration. This is not an unreasonable assumption at the scale of molecular flows. Most of the molecular movements that we describe above originate no more than two or three molecular mean free paths from the sensing surface—a sub-microscopic distance in human terms.

What we see as eddies and turbulence at the human scale is many orders of magnitude greater than the individual molecular movements on which our equations depend. Consequently, these macroscopic phenomena should be considered proper systems of study on their own merit, and not aberrations to the formulae.

## REFERENCES

INTERNAL REFERENCES: These are other papers in this collection that are either cited or linked in this paper. They also include such papers that especially relevant to the subject matter of this paper.

[ONE-DIMENSIONAL SPACE](#): Introduces the elegant and useful concept of one-dimensional space. This sole spatial dimension is a measure of proximity. Molecules are either moving closer to our object of interest or are moving farther away. The speeds at which they are doing either of these two things can be measured. All molecular parameters (save those dealing with wind) may be expressed in terms based upon this one-dimensional space.

[THE TWO POPULATIONS](#): Introduces the two populations of atmospheric molecules that acceptance of the second principle creates. These are the general population and the interacting population. This interacting population only comes into being when an object of interest is postulated, and exists only in relation to that object of interest.

[MOLECULAR NUMBERS](#): Definition of number density – Number density in ideal gases – Tables – Avogadro's Law – Number density in the general population – References.

[MOLECULAR MASSES](#): Variability of molecular masses – Gaseous abundances in dry air – Isotopic abundances – Mean molecular masses – Carbon 14 – Abundance of water vapor – Mean molecular impulse masses – Significance of mean impulse masses – References.

[MOLECULAR SPEEDS AND VELOCITIES](#): The mean proximity velocity – The root-mean-square velocity or standard deviation – Distribution of proximity velocities – Mean proximity velocities – Distribution of interaction velocities – Mean interaction velocities – Comparisons of velocities and equivalences – Mean speeds and velocities at selected temperatures for dry air – Mean speeds and velocities at selected temperatures for water vapor – References.

[MOLECULAR FLOWS](#): Flows in the general population – The imaginary tunnel – The Universal Flux Equation – The Equilibrium Flux Equation – Converse fluxes – Equilibrium molecular flows at selected temperatures for dry air – Equilibrium molecular flows at selected temperature for water vapor – References.

[MOLECULAR IMPULSES](#): Definition of impulse – The Universal Impulse Equation – Momentum and impulse in ideal gases – Impulse in real gases – Significance of impulse transfer – Molecular impulses in moving air – Molecular impulses at selected temperatures for dry air – Molecular impulses at selected temperatures for water vapor – References.

[KINETIC ENERGIES OF TRANSLATION](#): Definitions – Distributions and means for the  $p$  sub-population – Distributions and means for the  $i$  subset – Distributions and means for the true path modes – Comparisons and equivalences – Molecular kinetic energies in moving air – Molecular kinetic energies of translation for an ideal gas at selected temperatures – References.

[GAS LAWS IN THE FREE ATMOSPHERE](#): This paper shows how many of the gas laws useful and valid in the laboratory are neither when applied to the free atmosphere. Many gas laws require that the parcel of air be in a state of equilibrium. The atmosphere is never really in a state of equilibrium—certainly not when weather is occurring. Some gas laws require keeping one parameter constant while varying another. This is not possible in the free atmosphere.

[PROBABILITY DENSITY CURVE](#): The Probability Density Equation, the mean value, standard deviation, history of the Probability Density Curve, the Normal Curve, velocity distribution along a single Cartesian axis, velocity distribution along the proximity axis, the Error Function Curve, The Complimentary Error Function Curve, proximity velocity curves, Table of Selected Values, evaluating the error functions.

[THE NATURE OF ATMOSPHERIC TEMPERATURES](#) – Develops a universal definition of gas temperatures that can be applied to the real free atmosphere, and shows how the ideal gas definition is a special case of the universal equation.

[THE NATURE OF ATMOSPHERIC PRESSURES](#) – Develops a universal definition of atmospheric pressures that can be applied to the real free atmosphere, and shows how the ideal gas definition is a special case of the universal equation.

**EXTERNAL REFERENCES:** These are papers by other authors that contain statements or data that are specifically incorporated into the current discussion.

D. Tabor; **Gases, Liquids, and Solids**; Third Edition; Cambridge University Press, 1991.

Arthur Brown; **Statistical Physics**; Elsevier, New York, 1970.

James Jeans; **An Introduction to the Kinetic Theory of Gases**; Cambridge Library Collection, 1940.

<http://keisan.casio.com/exec/system/1180573449>. This is the web site used to calculate error functions.