STATISTICAL MECHANICS

Mechanics describes how physical bodies behave when subjected to various forces (the directions and speeds that two billiard balls will take following a collision, for instance). Statistics describes the most probable outcome of an event or a series of events, given certain assumptions (that the coin toss is fair, for instance). The combination of these two fields describes the most probable mechanical behavior of systems of very large numbers of individual molecules.

We will be using statistical mechanics in conjunction with kinetic gas theory to describe the most probable characteristics of both ideal gases (gases that exist only in the imagination of scientists) and the very real gases of our planet's atmosphere.

This endeavor is made both possible and remarkably accurate by what is called, "the law of large numbers". In what is perhaps its simplest form, the law states that the more samples that you take from a population, the more likely it is that your sample mean will equal the unknown true mean of the entire population (if you knew the true mean, you wouldn't need to take samples). That is, if you take a thousand samples, the mean of that sample set is more likely to equal the population mean than if you only took ten samples. Moreover, if you took a billion samples, then the likelihood that the sample mean equaled the true mean would be very much greater.

In fact, with a sample size that large, the probability that your sample mean would deviate significantly from the true mean would be very slight. As a scientist, you would be justified in assuming that your sample mean—rounded to three significant figures—is essentially the same as the population mean—rounded to three significant figures.

A note of caution—the more significant figures you use, the shakier your ground becomes. Valuations of three significant figures are the scientific norm for measurements, and you are rarely justified in using more than three for measurements. Your expression may be more precise, but it is not necessarily more accurate. With constants, on the other hand, you are justified in using as many significant figures and you can conveniently manage. Using constants having many significant figures increases both the precision and the accuracy of your results.

Let us take a cubic meter of an ideal gas with molecular characteristics that are the same as that of the mean for dry air. At normal temperature and pressure that system would contain an average of some 2.69×10^{25} molecules. Each of these molecules will undergo an average of some 5.44×10^{9} collisions each second. After each collision, the individual molecules direction of movement and speed is likely to change. That results in an average of 1.46×10^{35} speed changes per cubic meter of air per second.

Let us suppose that we wanted to know the mean molecular translational speed for that system of molecules. At the current state of scientific knowledge, we cannot measure that speed.

Statistical mechanics, however, permits us to calculate that average speed, based on certain assumptions. (If you're curious, it turns out to be some 447 meters per second.)

The number of events (1.46 x 10^{35}) is so huge that the chances of the calculated speed (447meters per second) differing significantly from the actual speed (which we cannot measure) is vanishingly small.

In practical terms, if you were to graph the theoretical speed distribution based on statistical mechanics on a sheet of paper, the chance that the true distribution would fall outside that thin line of ink describing the theoretical distribution would be virtually nil.

This is the justification for using the techniques of statistical mechanics to describe the behavior of ideal gases. As we shall see, we can also use these same techniques (with some obvious modifications) to describe the real gases of the atmosphere.

